Toward the AdS/CFT dual of the "Little Bang"

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December 13, 2011

Abstract

This (rather subjective) review sums up few years of work devoted to explain various aspects of high energy heavy ion collisions using the AdS/CFT correspondence. The central issue of is is formation of the trapped surface (black hole) phenomenon, seen by a distant observer as the entropy production. We end up discussing an issue of classical gravitational radiation by an ultrarelativistic falling body and the so called breaking self-force related to it.

1 Introduction

Spectacular conformation of the hydrodynamical predictions at RHIC, as well as very strong jet quenching (including heavy quarks) has lead to the "strongly coupled Quark-luon Plasma" (sQGP) paradigm, see e.g. [1]. These events put understanding of the strongly coupled regime of gauge theories phase at the front of several communities involved. For a time, discussion of the main nonperturbative phenomena – confinement, chiral symmetry breaking, topology – were put aside and the community focused on the deconfined/chirally high-T phase, trying to understand the difference between the weakly and strongly coupled plasmas.

While such examples as strongly coupled electromagnetic plasmas and cold fermionic gases at large scattering lengths has provided a lesson or two, a beautiful gift from the string community– the AdS/CFT correspondence – quickly became the major tool of the Nuclear Theorists, as well as a prime examples of the "string theory applications".

Another major event in the field – the first LHC heavy ion run, at the end of 2010 – have produced nice confirmations of old hydrodynamical predictions and even more spectacular display of jet quenching, with up to a 100 GeV dissipated into the plasma. An example of new theory development is "the second act of hydro": the calculation and observation of higher harmonics of the flow induced by initial fluctuations. As an example of how useful a collaboration between two theory communities may be, let me mention that behind our study of the higher flow harmonics [2] (not to be discussed below) would not be possible without the zeroth order conformal solution found by Gubser [3].

There is no need to go into detail into description of classic results obtained in the AdS/CFT framework [4], such as thermodynamics of strongly coupled N=4 plasma [5] and transport properties derived from the linearized hydrodynamics, resulting in the famous $\eta/s = 1/4\pi$ expression for the viscosity-to-entropy ratio [6] . Let me just mention another very significant achievement: a general derivation of the full nonlinear hydrodynamics with gradient expansion from the gradient expansion of

the Einstein equation[1] [7, 8, 9], based on the "membrane paradigm" for the black hole horizon developed in 1980's.

This is a very subjective review, based mostly on works with my (then) graduate student Shu Lin, as we were working our way into the AdS/CFT territory. As the reader will see, our main interests were related with the most difficult issues of QGP equilibration and jet quenching, related wit strongly-out-of-equilibrium situations. Our main idea was that equilibration in the AdS/CFT context has very simple intuitive meaning – objects simply fall into the black hole because of gravity forces – and the rest is just technical difficulties, which can be dealt with.

2 The worm-up: thinking about the Maldacena dipoles

Static pair of infinitely heavy fundamental quarks was the first ADS/CFT applications, and Maldacena's dipole calculation is so well known to anyone, that a reader may wander why do I want to write about it here. And yet, there are important lessons which are perhaps not well appreciated, perhaps warranting to come back to it at some later time.

It is hardly necessary to reminding the setting, as the reader was probably teaching it many times: a Nambu-Goto string pends due to gravity force into the 5-th dimension, like in the famous catenary (chain) problem. The celebrated Maldacena's result, the *Coulomb law at strong coupling* is

$$V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4}\frac{\sqrt{\lambda}}{L} \tag{1}$$

contains $\sqrt{\lambda}$ instead of λ in the weak coupling. (The numerical coefficient in the first bracket is 0.228, to be compared to the result from a diagrammatic re-summation below.)

What is the reason for this modification? For pedagogical reasons let me start with "naive guesses":
(i) Perhaps the strongly coupled vacuum acts like some kind of a space-independent dielectric constant, $\epsilon \sim 1/\sqrt{\lambda}$ which is reducing the effect of the Coulomb field, but similarly at all points.
(ii) Perhaps such dielectric constant is nonlinear effects, and thus is not the same at different points: but the fields created by static dipole are still just the electric field $\vec{E}$.

The AdS/CFT allows one to get many details about the problem and test such ideas: but we cannot get fields directly. Its rules allows one to find *holograms* on the boundary using bulk wave eqns for massless fields - which are scalars, vectors or gravitons. For example I will give our results for boundary stress tensor obtained from the (linearized) Einstein equation for the gravity perturbations calculated in [10], producing the *stress tensor* of matter $< T_{\mu\nu}(y) >$ at any point y on the boundary, induced by the Maldacena dipole. The solution is too technical to be presented here and even the resulting stress tensor expressions are too long. l The leading terms far from the dipole $y >> L$ is

$$T_{00} = \sqrt{\lambda}L^3\left(\frac{C_1 y_1^2 + C_2 y^2}{|y|^9}\right) f(\theta) \tag{2}$$

where C_1, C_2 are numerical constants whose values can be looked up in the paper and $f(\theta)$ is the angular distribution which is different that in the weak coupling dipoles, for which $T_{00} \sim \lambda L^2/y^6$. These and other results quickly put to rest simplistic ideas mentioned above: one clearly create more fields than just the electric one, with two static charges.

As we get glimpse of some first results from AdS/CFT we see that they are quite different from weak coupling and we would like to understand them. In fact we would like to do so both from the

[1] In the 19th century hydrodynamics was a very advanced theoretical field, teaching how to work with flows and sound waves. Stokes was clearly important for Maxwell, for example. Yet in 1970's, when I was starting my studies of QGP and of hydrodynamics of high energy collisions, most theorists were telling me those have no chance, being ridiculously simplistic and incompatible with quantum field theories. Apparently, not any more.

bulk (gravity) side as well as from the *gauge theory* side. It turns out the first is relatively easy. For example, both in the total energy and energy density we get $\sqrt{\lambda}$ because this factor is in front of the Nambu-Goto Lagrangian (in proper units). The reason the field decays as y^7 is extremely natural: in the AdS_5 space the function which inverts the Laplacean (analogously to Coulomb $1/r$ in flat 3d) has that very power of distance

$$P_s = \frac{15}{4\pi}\frac{z^2}{(z^2+r^2)^{\frac{7}{2}}} \tag{3}$$

with z being the 5-th coordinate of the source and r the 3-distance between it and the observation point[2].

In order to understand the same results from the gauge side we will need a bit of pedagogical introduction: the resolution will be given by the idea of *short color correlation time* by Zahed and myself [11]. In QCD, with its running coupling, higher-order effects modify the zeroth order Coulomb field/potential. Since we consider static problem, one can rotate time into Euclidean time $\tau = it$, which not only makes possible lattice simulations but also simplifies perturbative diagrams. Let us do Feynman diagrams directly in the coordinate representation. The lowest order energy, given by the diagram in which one massless quantum (scalar or gauge component A_0) is emitted by one charge at time τ_1 and absorbed by another at time τ_2 is just

$$V(L)(Time) \sim -g^2\int_0^{Time}\frac{d\tau_1 d\tau_2}{L^2+(\tau_1-\tau_2)^2} \sim -\frac{g^2(Time)}{L} \tag{4}$$

where 'Time' is total integration time and the denominator – the square of the 4-dim distance between gluon emission and absorption represents the Feynman propagator in Euclidean space-time. The propagation time for a virtual quantum $(\tau_1-\tau_2)\sim L$, thus the Coulomb $1/L$ in the potential.

Higher order diagrams include self-coupling of gluons/scalars and multiple interactions with the charges. A famous simplification proposed by 't Hooft is the large number of colors limit in which only *planar* diagrams should be considered. People suggested that as g grows those diagrams are becoming "fishnets" with smaller and smaller holes, converging to a "membrane" or string worldline: but although this idea was fueling decades of studies trying to cast gauge theory into stringy form it have not strictly speaking succeeded. It may still be true: just nobody was smart enough to sum up all planar diagrams[3].

If one does not want to give up on re-summation idea, one may consider a subset of those – the *ladders* – which can be summed up. Semenoff and Zarembo [12] have done that: let us look at what they have found. The first point is that in order that each rung of the ladder contributes a factor N_c, emission time ordering should be strictly enforced, on each charge; let us call these time moments $s_1 > s_2 > s_3$... and $t_1 > t_2 > t_3$.... Ladder diagrams must connect s_1 to t_1, etc, otherwise it is nonplanar and subleading diagram. Thus the main difference from the Abelian theory comes from the dynamics of the color charge itself! The (re-summed) Bethe-Salpeter kernel $\Gamma(s,t)$, describing the evolution from time zero to times s,t at two lines, satisfies the following integral equation

$$\Gamma(\mathcal{S},\mathcal{T}) = 1+\frac{\lambda}{4\pi^2}\int_0^{\mathcal{S}} ds\int_0^{\mathcal{T}} dt\frac{1}{(s-t)^2+L^2}\Gamma(s,t) \tag{5}$$

If this eqn is solved, one gets re-summation of all the ladder diagram. The kernel obviously satisfies the boundary condition $\Gamma(\mathcal{S},0)=\Gamma(0,\mathcal{T})=1$. If the equation is solved, the ladder-generated potential is

$$V_{\rm lad}(L) = -\lim_{T\to+\infty}\frac{1}{\mathcal{T}}\Gamma\left(\mathcal{T},\mathcal{T}\right)\ , \tag{6}$$

[2]Please recall that in gravity there are no cancellations between different contributions: any energy source perturbs gravity with the same sign.

[3]Well, AdS/CFT is kind of a solution, actually, but it is doing it indirectly.

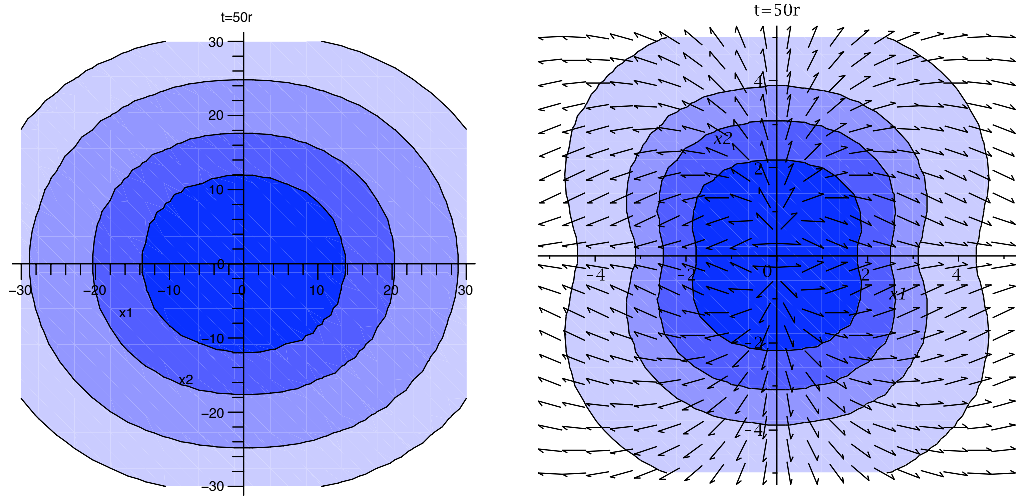

Figure 1: The hologram of a falling string. The contours show the magnitude of the energy density (a) and the Poynting vector T^{0i} in the transverse plane (b) at some time. The direction of the momentum flow is indicated by arrows.

In weak coupling $\Gamma \approx 1$ and the integral on the rhs is easily taken, resulting in the usual Coulomb law. For solving it at any coupling, it is convenient to switch to the differential equation

$$\frac{\partial^2 \Gamma}{\partial \mathcal{S}\, \partial \mathcal{T}} = \frac{\lambda/4\pi^2}{(\mathcal{S}-\mathcal{T})^2 + L^2} \Gamma(\mathcal{S}, \mathcal{T}) \ . \tag{7}$$

and change variables to $x = (\mathcal{S} - \mathcal{T})/L$ and $y = (\mathcal{S} + \mathcal{T})/L$ through

$$\Gamma(x, y) = \sum_m \mathbf{C}_m \gamma_m(x)\, e^{\omega_m y/2} \tag{8}$$

with the corresponding boundary condition $\Gamma(x, |x|) = 1$. The dependence of the kernel Γ on the relative times x follows from the differential equation

$$\left(-\frac{d^2}{dx^2} - \frac{\lambda/4\pi^2}{x^2+1}\right) \gamma_m(x) = -\frac{\omega_m^2}{4}\, \gamma^m(x) \tag{9}$$

For large λ the dominant part of the potential in (9) is from *small* relative times x resulting into a harmonic equation [12]

At large times $\mathcal{T}$, the kernel is dominated by the lowest harmonic mode. For large times $\mathcal{S} \approx \mathcal{T}$ that is small x and large y

$$\Gamma(x, y) \approx \mathbf{C}_0\, e^{-\sqrt{\lambda}\, x^2/4\pi}\, e^{\sqrt{\lambda}\, y/2\pi} \ . \tag{10}$$

From (6) it follows that in the strong coupling limit the ladder generated potential is

$$V_{\rm lad}(L) = -\frac{\sqrt{\lambda}/\pi}{L} \tag{11}$$

which has *the same parametric form* as the one derived from the AdS/CFT correspondence (1) except for the overall coefficient. Note that the difference is not so large, since $1/\pi = 0.318$ is larger than the exact value 0.228 by about 1/3. Why did it happened that the potential is reduced relative to the Coulomb law by $1/\sqrt{\lambda}$? It is because the relative time between gluon emissions is no longer $\sim L$, as in the Abelian case, but reduced to parametrically small time of relative color coherence $\tau_c \sim 1/L\lambda^{1/2}$.

3 "Holographic e^+e^- annihilation" and the absence of the jets

In 1976, three years after discovery of QCD, e^+e^- annihilation experiments at SLAC has seen the first indications for two jets, created by quark-antiquark pair moving away from each other. That lead to discussion of "string breaking" phenomena, phenomenological models of onset of confinement in hadronic collision, known as the Lund model (for B.Andersen and collaborators), whose descendants are used by experimentalists for this day.

AdS/CFT version of the holographic e^+e^- annihilation has been developed in two works by S.Lin and myself [13, 10]. Before we describe some of its results, let me mention another possible usage of those results. Some of the dreams high energy theorists have about possible discoveries at LHC or beyond is existence of "hidden" interactions which we don't see because the so called "mediators" – particles which are charged both under Standard Model and hidden theory – are heavy. Perhaps the hidden sector is strongly coupled: if so one may wander how production of a pair of new charges would look like. Further discussion of this issue (and more references) can be found in a paper by Hofman and Maldacena[14], who made further steps toward understanding the "strongly coupled collider physics".

The setting of Refs [13, 10] is simple: two heavy quarks *departing* from each other with velocities $\pm v$, on the boundary of the AdS$_5$ space. The AdS/CFT require charges connected by a classical string which (unlike the QCD one) : is not breaking but stretches indefinitely [4]. Before solving the corresponding equation in full, we will first discuss "scaling" solutions in the separable form

$$z(\tau, \eta) = \frac{\tau}{f(\eta)} \tag{12}$$

where τ, η are the proper time and space-time rapidity. The corresponding solution was obtained, but we found that it can only exists for sufficiently small[5] velocities. Moreover, the analysis of the classical stability of such scaling solution revealed that it gets unstable for $Y > Y_m \approx 1/4$, where quark rapidity is related to their velocity in the usual way $v = tanh(Y)$. For larger velocity of the quarks the scaling solution has to be substituted by a non-scaling one, depending on both variables in a nontrivial way, which was analyzed numerically.

The physical picture an observer at the boundary would see is again given by a (gravitational) hologram of the falling string calculated in the second paper [10]. One feature of the result should not be surprising for the readers who followed the description of the hologram of the *static* Maldacena string above: indeed, no trace of a string is in fact visible. The results, obtained after heavy numerical calculation – shown in fig.1 – produced a near-spherical explosion. This means that *there are no jets at strong coupling*!

Furthermore we found this explosion to be *non-thermal and thus non-hydrodynamical*, in the sense that the stress tensor found (although of course conserved and traceless) cannot be parameterized by the energy density and isotropic pressure. Indeed, thermal physics would require a black hole which is not at hand.

4 "Collapsing membrane" and the dynamics of the equilibration scale

The string is only one type of "bulk" objects which may fall into the AdS center. In relatively early paper Sin, Zahed and myself [16] first argued that exploding/cooling fireball, created in high energy

[4] This is true for classical string minimizing the action. However account for fluctuations of the string allows it to touch the flavor brane and break: in fact Peeters, Sonnenschein and Zamaklar [15] have calculated the holographic decays of large-spin mesons this way.

[5] In particular, for velocities going to zero one finds a strongly coupled version of Ampere's law, the interaction of two nonrelativistic currents.

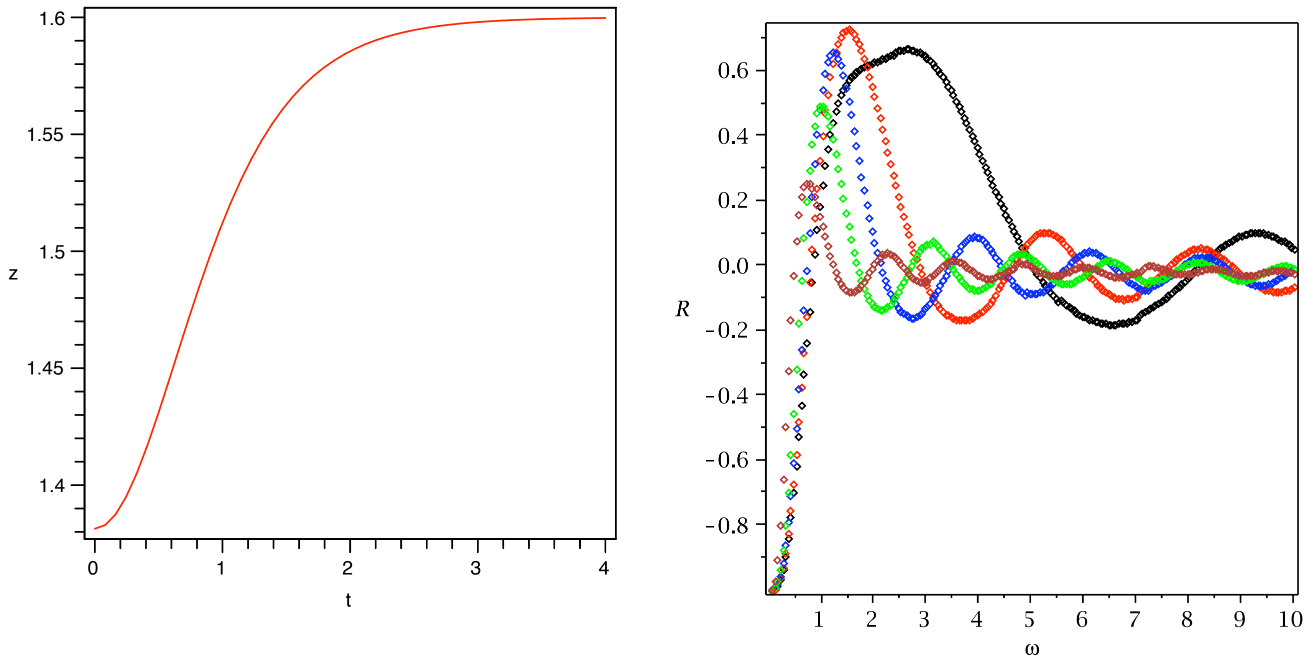

Figure 2: (a) The shell trajectory as a function of time. It starts at rest at $z = z_0$ with a constant acceleration, followed by a constant falling and eventually approaches the horizon in a exponential fashion. The parameter we choose are $\kappa_5^2 p = 1$ and $z_h = 1.6$. (b)The relative deviation R of the spectral density at zero momentum $q = 0$ at different stages of thermalization. The oscillations are explained by "echo" effect.

collisions, is a black hole formed by the collision debris and then falling toward the AdS center. While the specific solution discussed in that paper was spherically symmetric and thus more appropriate for cosmology than for heavy ion applications, it was a direct predecessor of series of papers by Janik and collaborators on "rapidity independent" falling horizons.

In this section we would start discussing the issue of equilibration, keeping to a picture that UV modes are fast and equilibrate quickly, while in IR the life is more slow and equilibration is delayed. Thus the idea of certain "equilibration front" appears, moving in scale from UV to IR. In AdS/CFT language this front can be thought of like some material objects, falling along the "scale" coordinate z.

The paper in which it has been worked out , by Lin and myself [17], considered the simplest geometry in which a shell (elastic membrane) is flat in x_1, x_2, x_3 and is collapsing *under its own weight* in AdS$_5$. The metric is simple: it is thermal AdS above the shell and empty AdS below. The only questions are what is the equation of motion of the shell and what can different observers see on the boundary (in the gauge theory).

An elegant element of the paper is the systematic use of the so called Israel junction condition. The falling velocity (in time of the distant observer t) is given by:

$$\frac{dz}{dt} = \frac{\dot{z}}{\dot{t}} = \frac{f\sqrt{(\frac{\kappa_5^2 p}{6})^2 + (\frac{3}{2\kappa_5^2 p})^2(1-f)^2 - \frac{1+f}{2}}}{\frac{\kappa_5^2 p}{6} + \frac{3}{2\kappa_5^2 p}(1-f)} \tag{13}$$

where $\kappa_5^2 p$ is 5-dim gravity constant and shell elastic constant and $f = 1 - z^4/z_h^4$ as usual, in thermal AdS. The value of the horizon position z_h, to which the shell asymptotically goes, see Fig.2 depends on the total mass of the shell (as detailed in the paper).

The main question is by which experiments an observer on the boundary can distinguish the true thermal state (thermal AdS metric) from "quasiequilibrium" one in our solution. A "one-point observer"

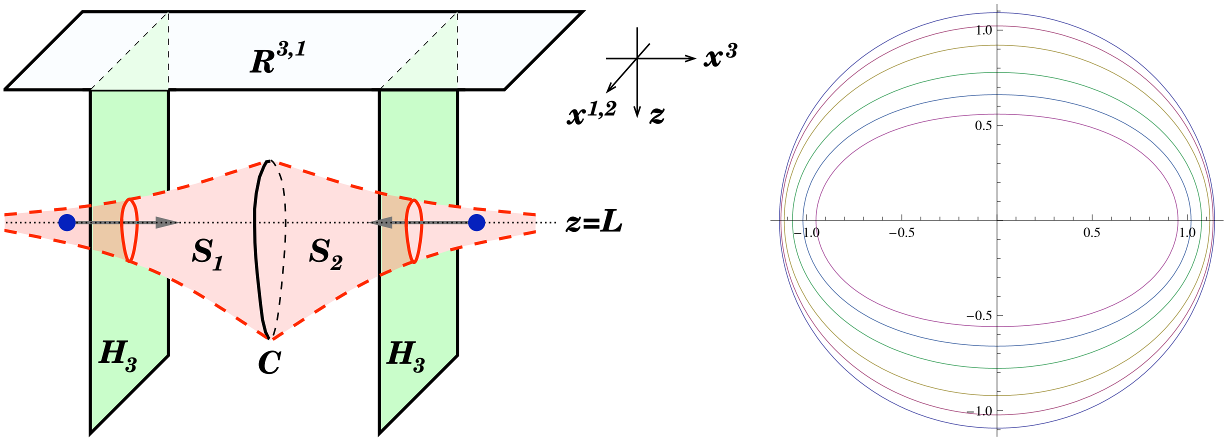

Figure 3: (a) From [18]:A projection of the marginally trapped surface that we use onto a fixed time slice of the AdS geometry. The size of the trapped surface is controlled by the energy of the massless particles that generate the shock waves. These particles are shown as dark blue dots. (b) From [19].The shapes of the trapped surface at at collision time in the transverse plane, for impact parameters $0.4L$, $0.6L$, $0.8L$, $1.0L$, $1.1L$, $1.14L$ from the outer to the inner, the innermost being the critical trapped surface.

would simply see the equilibrium pressure and energy density, as the metric above the shell is thermal AdS. Yet more sophisticated "two-point observers" can measure correlation functions, and they will see deviations from the equilibrium because a signal can penetrate *below* the shell. Solving for various two-point functions in the background with falling shell/membrane we found such deviations: they are oscillating in frequency around thermal ones. The corresponding oscillation is explained by certain "echo" times for a signal reflected from the shell: see more on this interesting phenomenon in [17].

5 Entropy production and the trapped surfaces

Multiple attempts over the years to understand the initial equilibration of QGP and provide quantitative theory of the entropy production has not yet succeeded. The so called *"glasma"* approach, treating random gluonic fields via classical Yang-Mills equations, has been pursued by many groups. Related phenomenology tells us that the typical momenta of those gluons, known as the "saturation scale" Q_s, is at RHIC/LHC in the range of 1.5/2 GeV. This is uncomfortably close to their momenta in the sQGP at hydrodynamical stage to follow, the "most perfect liquid" as we now know, and one may wander if the perturbative (weak coupling) approach currently used is indeed justified.

General relativity provides a remarkably simple mechanism of the entropy production: as the information happened to be surrounded by the "trapped surface" cannot be retrieved, it is the entropy produced! Even more convenient fact is that the trapped surfaces do not respect the usual causality arguments: they can form in the perfectly empty space, in *anticipation* of the collision to follow later, as they are always justified *a posteriori*. This leads to a paradoxical conclusion of some entropy being there already at t=0, as one indeed finds the trapped surface being there already.

The first step had been done by Gubser, Pufu and Yarom [18], who proposed to look at heavy ion collision as a process of head-on collision of two point-like black holes, separated from the boundary by some depth L see Fig.3(a). By using global AdS coordinates, these authors argued that (apart of obvious axial O(2) symmetry) this case has higher – namely O(3)– symmetry with the resulting black

hole at the collision moment at its center, thus in certain coordinate

$$q = \frac{\vec{x}_\perp^2 + (z-L)^2}{4zL} \tag{14}$$

the 3-d trapped surface C at the collision moment should be just a 3-sphere, with the radius q_c. (Here $x_\perp$ are two coordinates transverse to the collision axes.) The picture of the surface, taken from their paper, is shown in Fig.??. One can find q_c and determine its relation to CM collision energy and Bekenstein entropy. For large q_c these expressions are just

$$E \approx \frac{4L^2 q_c^3}{G_5}, \;\; S \approx \frac{4\pi L^3 q_c^2}{G_5}, \tag{15}$$

from which, eliminating q_c, the main result of the paper follows, namely that the entropy grows with the collision energy as

$$S \sim E^{2/3} \tag{16}$$

Note that this power is in general $(d-3)/(d-2)$ directly relates to the d=5-dimensional gravity, and is different from the 1950's prediction of Fermi/Landau who predicted the power 1/2. This prediction is also not far from reality[6]

The generalization to non-central collisions has been done by Lin and myself [19]. There is no point in going into technical details here, so let me describe the equations in words. They resemble electrostatic problem with two charges inside the metallic cavity, except that there is an extra boundary condition on the fields (from smoothness of the surface) and, last but not least, the surface is not given but has to be determined. The way to solve the problem was via rewriting it as integral equation.

From gravity point of view the qualitative trand was clear: two colliding objects may merge into a common black hole only provided that the impact parameter is less than some critical value $b_c(E)$, depending on the collision energy. Indeed, with b rising, the trapped energy decreases and angular momentum increases, so at some point no black hole can be formed. Interestingly, it happens as a jump, just a bit below this impact parameter reasonable trapped surface and black hole exist and nothing obvious indicates that at large b none is formed. The shape of the trapped surface is shown in Fig.3(b), and the dependence of the trapped surface area on impact parameter is shown in Fig.4, from next paper, comparing our points with a series of curves later obtained by Gubser, Pufu and Yarom [20].

From the point of view of the gauge theory this behavior is however a complete surprise: it predicts first order transition as a function of impact parameter, with creation of thermal fireball with significant entropy at $b < b_c$, while no such things is there for $b > b_c$. (Needless to say, classical gravity and such jumps are consequences of the large N_c approximation, presumably smoothened out at finite N_c.) As discussed in our paper [19], the experimental multiplicity data from PHOBOS experiment at RHIC shown in Fig. suggest a relatively sharp transition from "heavy-ion-like" to "pp-like" entropy as a function of impact parameter, but the experiments have not yet addressed the transition from one to another well enough, to make this comparison quantitative.

In the same paper [19] we have pointed out that the simplest geometry to study the trapped surface would be wall-wall collision, in which there is no dependence on transverse coordinates x^2, x^3 and thus a sphere becomes just two points in z, above and below the colliding bulk objects. We elaborated on this in more recent work [21], considering collision of two walls made of material with different "saturation scales" (e.g. made of lead and cotton) and studied conditions for trapped surface formation.

(In connection with this, let us note that some papers consider collisions of walls which have no source in the bulk and are $\sim \delta(x \pm t) z^4$ perturbation of the metric. Although such walls may have the

[6]Simplistic comparison with total multilicity ignores the nonzero baryon number, important especially at lower collision energies, which is not produced and should be removed.

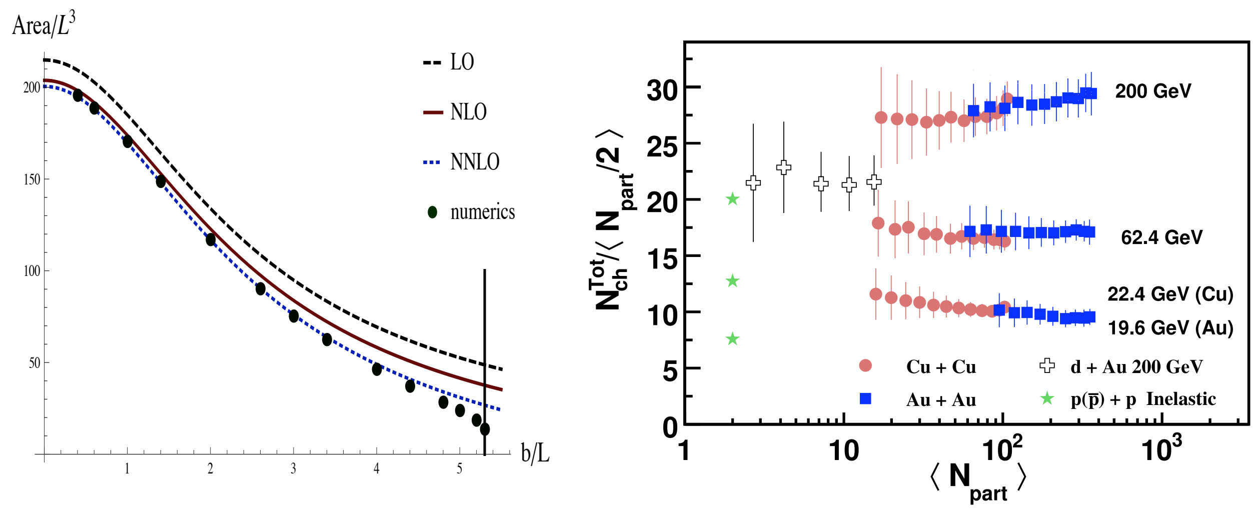

Figure 4: (a) The area of the trapped surface versus impact parameter, with the comparison of the numerical studies by Lin and myself [19] shown by points and analytic curves from [20]. (b) Experimental data from PHOBOS collaboration, on the charge particle multiplicity at RHIC per participant nucleon. Three sets of data, one above the other, are for three collision energies, 200, 62.4 and 19.6/22.4 GeV. All three show values independent on centrality of the collision (left peripheral, right central) for AuAu collisions (closed points), which is different and higher if compared to pp collisions (green stars) and dAu collisions (open squares) in which no QGP is formed.

same boundary hologram as ours, they lead to mathematical inconsistencies. They cannot be treated as perturbations at large z, and their trapped surfaces are not closed from below, and thus their area is not even defined.)

6 The resummed hydrodynamics and the entropy at LHC

Not all the entropy is produced promptly: some is produced by the dissipative effects at the hydrodynamical stage. In order to estimate this amount one first need to specify what exactly is meant by "hydro" and by its "start". To define a starting moment is relatively easy: any theory may be considered valid as long as it works with some preset accuracy (say, one percent). hydro itself comes at least in four forms: (i) "ideal hydrodynamics" without dissipation, (ii) Navier-Stokes (NS) which includes the viscous term, (iii) the "second order" hydro with the second gradients of some kind; (iv) the "resummed" hydro suggested at least for AdS/CFT in [22]).

On a theory side, there has been a significant progress in solving Einstein equation for "gravitational collisions", see e.g. [23]. We will not go into this vast subject here, and only focus on the central issue of equilibration and onset of hydrodynamics. A very interesting study has been recently performed in Ref. [24]. In the rapidity-independent approximation, these authors had follow evolution for ~ 20 different and arbitrary initial conditions, and study how they equilibrate. Fig.5(left), from this paper, show convergence of all of those evolutions to some universal function of the variable $w = \tau T$

$$\frac{dw}{d\ln\tau} = F(w)\,, \tag{17}$$

whose existence is the essence of the "resummed hydro" proposed in [22]). As one can see, depending on accuracy, on may assign the beginning of hydro to some "initial" $w_i = 0.4.. - .6$. The plot on the right demonstrate that at such time the anisotropy is still large and viscosity is important.

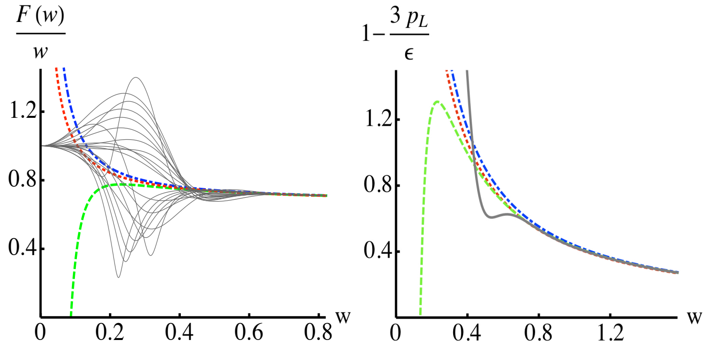

Figure 5: (left) The temperature evolution combination $dlog(w)/dlog\tau$ for different initial conditions (black thin curves) converging into a universal function of $w = T\tau$, compared to hydro. (right) The pressure anisotropy for one of the evolutions compared to 1-st (NS), 2-nd and 3-ed order hydrodynamics.

The issue got into focus recently due to one of the first discoveries made at the first LHC PbPb run. Indeed, it was found [25] that the multiplicity in PbPb collisions grow with energy more rapidly than in pp:

$$\frac{dN_{ch}^{PbPb}}{dy}(y=0,s) \sim s^{0.15} \qquad \frac{dN^{pp}\_ch}{dy}(y=0,s) \sim s^{0.11} \tag{18}$$

and the absolute value is 30-40% larger than the CGC estmates calibrated by pp data. In other words, From the RHIC energy ($E = 0.2\,TeV$) to the LHC, the double ratio is

$$\frac{\frac{dN}{d\eta}|_{PbPb,LHC} \;/\; \frac{dN}{d\eta}|_{pp,LHC}}{\frac{dN}{d\eta}|_{AuAu,RHIC} \;/\; \frac{dN}{d\eta}|_{pp,RHIC}} = 1.23\,. \tag{19}$$

This noticeable change with the energy calls for a theoretical explanation.

Lublinsky and myself [26] recently address both issues. In short, we propose a simple form for the function $F(w)$ and calculate the entropy produced, from the time w_i on. It turns out to be about 30%. Furthermore, we get the following expression for the contribution to this double ratio $\approx 1 + \frac{3[\bar\eta(LHC)-\bar\eta(RHIC)]}{2w_i+3\bar\eta(RHIC)}$ and show, that the observed growth can be naturally explained by the viscosity growth, from RHIC to LHC, predicted by a number of phenomenological models.

7 Jet quenching and the longitudinal "self-force"

There is no place here to review the well known works which established jet quenching for heavy and light quark jets: it is clearly done in other reviews of this volume. Focusing on the latter case, it is sufficient to say that those basically describe the quenching process by falling of some massive object in radial AdS coordinate, toward the AdS center (or into IR). "Quenching" is not seen in the bulk, as 4-momentum of the falling object is conserved, but the holographic pictures at the boundary do show Mach cones in very good agreement with hydro predictions.

This article is also not a place to provide a review of jet quenching phenomenology. Let me just comment that RHIC reconstructed jets and especially dijet LHC data from the first PbPb run have significantly changed the views of its mechanism. Huge losses of the associate jets, reaching $O(100\,GeV)$, together with basically unchanged narrow distribution in jet relative angle at π, point to robust longitudinal breaking force rather than transverse kicks predicted perturbatively. Its magnitude apparently way exceed the tensions of the QCD string , and probably even that of the AdS strings on which the abovementioned heavy quark quenching theory has been based.

So, what may the physical nature of this longitudinal force be? Recent work by Yee, Zahed and myself [29] suggests to look at next-order bulk processes including graviton/dilaton radiation of the "falling body". Its non-AdS (weak coupling) predecessors includes my 1973 paper with Khriplovich [30] in which electromagnetic and gravitational radiation for ultrarelativistic particle in gravity background has been calculated, as well as the 2002 paper with Zahed [31] in which we discussed classical QCD syncroton-like radiation in transverse color field.

The calculation of gravitational radiation from an ultrarelativistic particle moving in a curved background is a notoriously difficult problem. However, a local version of the expression for self-force has been derived [28, 27] who, in gravittional setting, obtained a remarkable contribution to the "self-force"

$$a^a = mu^b u^c \int_{-\infty}^{\tau^-} d\tau' u^{a'} u^{b'} (\frac{1}{2}\nabla^a G_{bca'b'} - \nabla_b G_c{}^a{}_{a'b'} - \frac{1}{2}u^a u^d \nabla_d G_{bca'b'}) \tag{20}$$

in which the integral is done over proper time and the past world line of the particle till regulated present time τ^-, and G is the retarded Green function for the Einstein equation with the particle as the source. Note that the bracket is just the Chrystoffel force for a gravity perturbations, induced by the past history of the particle itself.

Returning to electrodynamics in flat 3+1 dimensional space one finds that there is no such local expression: and furthermore, there cannot be, as the retarded propagator is $\sim \delta(x^2)$ and localized on the light cone, never intersecting the particle path in the past. Yet we observed that it is not true in odd space-time dimensions, and calculated (in a separate companion paper) the radiation and self-force in 2+1 and 4+1 space-times: the results match the formula.

Encouraged by it, we calculated the self-force using the small-time expansion of the retarded propagator. Many structures vanish because covariant acceleration (and higher derivatives) are zero on the geodesic. Also thermal AdS has simplified Ricci tensor, proportional to the metric itself, thus $R_{ij}\dot{x}^i\dot{x}^i \sim \gamma^0$, not the square of the jet gamma factor γ^2 as one would naively expect. But the quadratic Riemann term is such that all indices of the 4 velocities generate the maximal power of γ. The resulting breaking force

$$m\ddot{x}^a \approx -\frac{G_5 m^2}{30\pi}\left(\int d\epsilon\right) \mathbf{R}^m{}_e{}^n{}_b \mathbf{R}_{mcnd}\, \dot{x}^e \dot{x}^b \dot{x}^c \dot{x}^d\, \dot{x}^a, \tag{21}$$

where the small parameter $\epsilon \sim 1/\gamma^2$ due to acceleration-induced curving of the path. The relativistic force thus is[7] $\sim \gamma^3$. The 5-dim gravitational constant is small, $G_5 \sim 1/N_c^2$ yet power of the jet gamma factor is large. Not only this self-force seem to be very large, it is also rapidly grow with the distance travel, stopping the jet earlier than the geodesic fall would predict. The issue of self-force and its relation to the gravitational radiation in thermal AdS are rather new, and should of course be studied further, before ay phenomenological applications can be discussed.

8 Summary

While AdS/CFT correspondence does not hold for QCD and it is not good to predict particular numbers, it clearly is hugely successful in providing correct qualitative picture of strongly coupled dynamics. It is also constantly forcing the theorists to look at the familiar problems in a completely new light. Naturally, as we learn more and more of it, the problem discussed are becoming more complex. Many efforts are going into more and more "realistic" solutions for ultrarelativistic gravitational collisions, the dual to the "Little Bang" we hope to find.

[7]Thus the usual non-relativistic force is $O(\gamma^2)$, same power as in Schwartzschield metric [30].

Let me end by reiterating few applications/conclusions/questions discussed above.

"Maldacena dipole" is next simplest field configuration, after that of one charge. And while we know scalar and stress tensor picture of it, we still have no idea what fields are involved and what shape they have. Early works on trying to resum certain diagrams to reproduce the strong coupling answers were made but not followed.

"No jets in e^+e^- at strong coupling ": heavy quarks are always connected by the string, but its falling into the 5-th dimension nontrivially depend on the velocity, and it is so rapid that there is no trace of a string left in the resulting hologram on the boundary.

"Collapsing membrane" describes time-independent relaxation process, in which relaxation proceed from UV to IR modes. Elegantly it is described by a membrane falling into IR under its own weight. The scale , being the z position of the membrane, changes as prescribed by the equation following from Israel junction condition. While "single-point-observer" finds matter apparently thermal, the "two-point-observed" are able to look under the membrane and find the non-equilibrium modifications.

"Trapped surfaces" can be instantly formed in the collisions, providing estimates (from below) of the produced entropy. Remarkably, as a function of impact parameter the solution disappear abruptly. This correlates well with relatively rapid switch from "no hydro" to "full hydro" regimes in experiment, as the impact parameter varies.

"Universal hydro" regime can be studied by colliding objects with various initial distributions in scales. We gave arguments that LHC entropy growth in PbPb collisions may be due to anticipated changes in viscosity.

Gravitational radiation self-force is new and rather difficult thing to understand/calculate. It remains unknown how our estimates correlate with the actual intensity of bulk gravitational radiation. Yet it seems clear that effects which are formally suppressed by power of N_c can still be perhaps dominant in practice, due to very large jet Lorentz factor.

But truly important would be to build a bridge between weak and strong coupling limits, which look at the moment disconnected. This is why I included our discussion of Coulomb resummation, Another fascinating possibility is establishment of the correspondence between fantastic progress in perturbative analysis of $\mathcal{N}$=4 scattering amplitude and their strong coupling limits, derivable from AdS/CFT.

9 Acknowledgements

My understanding of AdS/CFT applications, at whatever depth it is, came from multiple discussion with many people, but mostly with my Stony Brook collaborators Ismail Zahed, Shu Lin and Derek Teaney. The work is partially supported by the US-DOE grant DE-FG03-97ER4014.

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